

(PMAT 101)  
SRR & CVR Government Degree College (Autonomous): Vijayawada  
JULY 2021-BATCH 2020-21  
Department of Mathematics  
Real Analysis  
Semester -I: End Examination

Time: 3 Hours

Max. Marks: 60

Part -A

I. Answer all Multiple Choice Questions (MCQ).

15x1=15M

- 1) Let  $f: [a, b] \rightarrow \mathbb{R}$  be a bounded function such that  $m$  is the lower and  $M$  is the upper bound of  $f$  then the relation  $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$  is  
a) True                      a) False
- 2)  $f \in R(\alpha)$  on  $[a, b]$  iff for every  $\epsilon > 0 \exists$  a partition  $p \ni U(p, f, \alpha) - L(p, f, \alpha)$   
a)  $< \epsilon$     b)  $> \epsilon$     c)  $= \epsilon$     d) None
- 3) If  $f$  is monotonic on  $[a, b]$  and if  $\alpha$  is continuous on  $[a, b]$ , then  
a)  $f \in R(\alpha)$     b)  $f \notin R(\alpha)$     c) None
- 4) The Dirichlet function is defined by
$$f(x) = 1 \text{ if } x \neq 0$$
$$= 0 \text{ if } x = 0$$
  
a)  $f$  is unbounded on  $\mathbb{R}$     b)  $f$  is bounded on  $\mathbb{R}$     c)  $f$  is R-S integrable on  $\mathbb{R}$     d) None
- 5)  $\lim_{n \rightarrow \infty} \frac{1}{1+nx} =$   
a) 1    b) 0    c) does not exist    d) None
- 6) The series  $\sum_{n=1}^{\infty} \frac{1}{n^p + n^q x^2}$  converges uniformly if  
a)  $p=1$     b)  $p < 1$     c)  $p > 1$     d) None
- 7) Every point wise convergent sequence of functions converges uniformly  
a) True    b) False
- 8) Improper integral  $\int_0^1 x^n dx$  is convergent for  
a)  $n > -1$     b)  $n < 1$     c)  $n=1$     d)  $n \neq 1$
- 9) The improper integral  $\int_0^1 x^{m-1}(1-x)^{n-1} dx$  exists iff  $m$  and  $n$  are  
a) +ve    b) -ve    c) either +ve or -ve    d) None
- 10)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x \sin(x^2+y^2)}{x^2+y^2}$  is  
a) 1    b) -1    c) 0    d) None

- 11) Every differentiable function is continuous  
 a) True b) False
- 12) A mapping  $f: x \rightarrow y$  where  $x$  and  $y$  are metric spaces, is continuous on  $x$  iff for every open set  $v$  in  $y$ ,  $f^{-1}(v)$  is ----- in  $x$   
 a) Open b) closed c) not defined d) None
- 13) Let  $f$  be a real valued continuous mapping defined on a metric space  $X$ . Let  $z(f) = \{x \in X \mid f(x) = 0\}$  then  $z(f)$  is  
 a) Open b) closed c) continuous d) discontinuous
- 14)  $E$  is a dense subset of  $X$  if  $E \cup E' =$   
 a)  $E$  b)  $E'$  c)  $X$  d) None
- 15)  $g(x) = \frac{x^2}{1+x^2} \forall x \in E$  is  
 a) Continuous b) Discontinuous c) undefined d) None

### Part -B

#### II. Answer ALL Questions

5x9=45M

16. a. Let  $X$  and  $Y$  be metric spaces with metrics  $d$  and  $\rho$  respectively. Let  $E$  be a subset of  $X$  and let  $p$  be a limit point of  $E$ . Let  $f: E \rightarrow Y$  be a mapping, then  $\lim_{x \rightarrow p} f(x) = q$  iff  $\lim_{n \rightarrow \infty} f(p_n) = q$  for every sequence  $\{p_n\}$  in  $E$  such that  $p_n \neq p$  and  $\lim_{n \rightarrow \infty} p_n = p$ .

(or)

b. Let  $f$  be a continuous mapping defined from a metric space  $X$  into a metric space  $Y$ . If  $E$  is a connected subset of  $X$ ; then  $f(E)$  is a connected subset of  $Y$ .

17. a.)  $f \in R(a, b)$  iff for every  $\epsilon > 0 \exists$  a partition  $p$  such that  $U(p, f) - L(p, f) < \epsilon$ .  
 (or)

b. Let  $\gamma$  be a curve on  $[a, b]$  such that  $\gamma^1$  is continuous on  $[a, b]$ , then  $\gamma$  is rectifiable and  $L(\gamma) = \int_a^b \|\gamma^1(t)\| dt$ .

18. a. A sequence  $\{f_n\}$  of the functions defined on a set  $E$  converges uniformly on  $E$  iff for every  $\epsilon > 0, \exists$  positive integer  $N$  such that  $|f_n(x) - f_m(x)| \leq \epsilon \forall m, n \geq N$  and  $x \in E$ .

(or)

b) Show that the metric space  $C(X)$  is complete.

19. a.) Test for the convergence of the improper integral  $\int_1^2 \frac{x \, dx}{\sqrt{x-1}}$

(or)

b.) Examine the convergence of the integral  $\int_1^{\infty} \frac{dx}{x\sqrt{x^2+1}}$ .

20) a. Show that  $\lim_{(x,y) \rightarrow (0,0)} xy \frac{x^2-y^2}{x^2+y^2} = 0$ .

(or)

b.) Examine the extreme value of the function  $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ .

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Part A

I. Answer all multiple choice questions:  $15 \times 1 = 15$  Mark

1) The General solution of the differential equation

$$(D^2 - 2)y = 1 \quad \text{is}$$

a)  $C_1 e^{\sqrt{2}x} + C_2 e^{-\sqrt{2}x}$

b)  $(C_1 + x C_2) e^{\sqrt{2}x}$

c)  $C_1 e^{\sqrt{2}x} + C_2 e^{-\sqrt{2}x} - \frac{1}{2}$

d) None of these.

2)  $\frac{1}{D^2 - 3} \cos x = \underline{\hspace{2cm}}$

a)  $\frac{1}{4} \sin x$

b)  $\frac{1}{2} \cos x$

c)  $\frac{1}{4} \cos x$

d)  $\frac{1}{2} \sin x$

3) The P.I. of  $y'' + y = \sin 4x$  is  $\underline{\hspace{2cm}}$

a)  $-\sin 4x$

b)  $-\frac{\sin 4x}{15}$

c)  $-\frac{\sin 4x}{10}$

d) None

4) The solution  $y_1(x), y_2(x)$  are said to be L.I.

If a)  $W(y_1, y_2) = 1$

b)  $W(y_1, y_2) = 0$

c)  $W(y_1, y_2) \neq 0$

d) None

5) What is Wronskian of  $1, e^x$ .

a)  $e^x$

b)  $1 + e^x$

c) 0

d) None

Find the general solution to  $y''' - y'' + y' - y = 0$

a)  $y = \frac{1}{2}e^t \sin 2t + \frac{1}{3}e^t \cos 2t$       b)  $y = \frac{1}{1}e^{-t} + \frac{1}{2} \sin t + \frac{1}{3} \cos t$

c)  $y = \frac{1}{1}e^t + \frac{1}{2} \sin 2t + \frac{1}{3} \cos 2t$       d)  $y = \frac{1}{1}e^t + \frac{1}{2} \sin t + \frac{1}{3} \cos t$

7) For a non-negative real constant  $n$ , an equation of the form  $(1-x^2)y'' - 2xy' + n(n+1)y = 0$  is called

- a) Legendre's equation      b) Legendre's polynomial  
c) Bessel's Equation      d) None of these

8) The value of Legendre Polynomial  $P_2(x)$  is

- a) 0      b)  $\frac{1}{2}(3x^2 - 1)$       c) 1      d)  $\frac{1}{2}(5x^3 - 3x)$

9)  $J_{\frac{3}{2}}(x) = \text{---}$

- a)  $\sqrt{\frac{2}{\pi x}} \sin x$       b)  $\sqrt{\frac{2}{\pi x}} (\sin x - \cos x)$       c)  $\sqrt{\frac{2}{\pi x}} \cos x$       d) None

10) The set which consists of more than one equations is classified as

- a) system of equations      b) system of variables  
c) system of constants      d) system of coefficient.

11) Trace of a Matrix is defined as

- a) sum of rows      b) sum of columns  
c) sum of all elements      d) sum of diagonal elements.

Solve the initial value problem  $y'' - 4y' - 5y = 0$

for  $y(0) = 0$  and  $y'(0) = 6$

a)  $y = e^{5t} - e^{-t}$

b)  $y = -e^{-5t} - e^{-t}$

c)  $y = 5e^{5t} - e^{-t}$

d)  $y = e^{3t} + 3e^{-3t}$

13) For the initial value problem

$$x(x-1)y''' - 3xy'' + 6x^2y' - \cos x(y) = 0, \quad y(x_0) = 1,$$

$$y'(x_0) = 0, \quad y''(x_0) = 7.$$

Which of the following intervals containing  $x_0$

do not guarantee the existence of a unique solution?

14) Which of the following ODE's satisfies the sufficient condition for the local existence and uniqueness theorem at  $x=0$ ?

a)  $x = x^{1/3}$

b)  $x = \sqrt{x}$

c)  $x = |x|$

d)  $x = x \log x$

15) Consider the IVP  $y' = 2y$ ,  $y(0) = 1$  then by

Picard's method, the  $n^{\text{th}}$  iterate is

a)  $\sum_{i=1}^n \frac{(2x)^i}{i!}$

b)  $\sum_{i=1}^n \frac{x}{i!}$

c)  $\sum_{i=1}^n \frac{2x}{i!}$

d) None

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N. Answer any one question out of two questions from each unit. 5 × 9 = 45 Marks

16) a) Solve  $(D^4 + 2D^2 + 1)y = x^2 \cos x$

(02)

b) Consider the equation  $y''' - 4y' = 0$ . Compute three L.I. solutions and wronskian of the solutions. find  $\phi$  satisfying  $\phi(0) = 0$ ,  $\phi'(0) = 1$ ,  $\phi''(0) = 0$

17) a) Solve  $y'' + ay = x^2 e^{3x}$  by annihilator method

(03)

b) Compute the solution of non-homogeneous equation,  $y''' + y'' + y' + y = 1$ , satisfying  $\psi(0) = 0$ ,  $\psi'(0) = 1$ ,  $\psi''(0) = 0$

18) a) State and prove Orthogonality property of Legendre polynomials.

(05)

19) b) S.T.  $\frac{d}{dx} (J_0(x)) = -J_1(x)$ ,  $\frac{d}{dx} (x J_1(x)) = x J_0(x)$

19) a) Determine exponential  $e^{At}$  for the system  $x' = Ax$

Where  $A = \begin{bmatrix} \alpha_1 & 0 & 0 \\ 0 & \alpha_2 & 0 \\ 0 & 0 & \alpha_3 \end{bmatrix}$  where  $\alpha_1, \alpha_2, \alpha_3$  are scalars.

(03)

b) Consider the system  $x' = Ax$  where  $A = \begin{bmatrix} -3 & 1 & 0 \\ 0 & -3 & 1 \\ 0 & 0 & -3 \end{bmatrix}$

Then show the matrix  $\phi(t) = \begin{bmatrix} e^{-3t} & te^{-3t} & \frac{1}{2}e^{-3t} \\ 0 & e^{-3t} & te^{-3t} \\ 0 & 0 & e^{-3t} \end{bmatrix}$

is a fundamental matrix.

(a) State and prove existence theorem by Picard's method.

(b)

b) Apply Picard's method to the initial value problem  $y' = xy$ ,  $y(0) = 1$ .

~~x = x = x~~

**(PMAT 102)**  
**M.Sc (Mathematics) P.G Examinations-July 2022**  
 SEMESTER-I  
**ORDINARY DIFFERENTIAL EQUATIONS**

TIME: 3Hrs

MaxMarks:60

**SECTION-A**

Answer any **FIVE** of the following. Each question carries 4 Marks. **5 x 4 = 20 M**

1. Solve  $xy^1 + y = x^4y^3$ .
2. Show that  $y = c_1e^x + c_2e^{2x}$  is the general solution of  $y^{11} - 3y^1 + 2y = 0$  on any interval, and find the particular solution for which  $y(0) = -1$  and  $y^1(0) = 1$ .
3. Using Annihilator method, find a particular solution of the d.e  $y^{11} + 4y = \cos x$ .
4. Verify that  $y = c_1x^{-1} + c_2x^5$  is a solution of  $x^2y'' - 3xy' - 5y = 0$ .
5. State the generating function of the Legendre's polynomials.  
Show that  $P_n(1) = 1$ .
6. Find  $P_0(x)$ ,  $P_1(x)$  and  $P_2(x)$  using Rodrigue's formulae.
7. Find the general solution of the system  $\frac{dx}{dt} = x + y$ ,  $\frac{dy}{dt} = 4x - 2y$ .
8. Find  $e^{At}$  when  $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ .
9. Show that  $f(x, y) = y^{(1/2)}$  does not satisfy Lipschitz condition on the rectangle.
10. Solve the Initial Value Problem (IVP)  $y^1 = -y$ ,  $y(0) = 1$  by the method of successive approximations.

## SECTION-B

Answer ALL questions. Each question carries 8 Marks.

5 x 8 = 40 M

(1a) State and prove Existence theorem.

(OR)

b) Suppose that ' $\varphi$ ' is a function with continuous derivatives and  $0 < x \leq 1$ .

Then  $\varphi^1(x) - 2\varphi(x) \leq 1$  and  $\varphi(0) = 1$ . Show that  $\varphi(x) \leq \frac{3}{2}e^{2x} - \frac{1}{2}$ .

(2a) If  $y_1(x)$  and  $y_2(x)$  are two solutions of a equation  $y'' + P(x)y' + Q(x)y = 0$  on  $[a, b]$ . Then prove that they are linearly dependent on this interval if and only if the Wronskian is identically zero.

(OR)

b) If  $y_1(x) = x$  is the solution for the differential equation  $x^2y'' + xy' - y = 0$ , then find the general solution of the differential equation.

(3a) Derive Rodrigue's formula for Legendre's polynomials.

(OR)

b) Prove that i)  $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$  ii)  $J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \sin x$ .

(4a) If  $W(t)$  represents wronskian of two nontrivial solutions of  $x' = a_1(t)x + b_1(t)y$ ,  $y' = a_2(t)x + b_2(t)y$  on  $[a, b]$ , then show that  $W(t)$  is either identically equal to zero or nowhere zero on  $[a, b]$ .

(OR)

b) Determine exponential  $e^{At}$  for the system  $X' = AX$  where  $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}$ .

(5a) State and prove Picard's theorem.

(OR)

b) Show that the function  $f(t, x) = e^t \sin x$ ;  $|x| \leq 2\pi$ ,  $|t| \leq 1$  satisfies the Lipschitz's condition.

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(PMAT 103)  
SRR & CVR Government Degree College (Autonomous): Vijayawada  
JULY 2021-BATCH 2020-21  
Department of Mathematics  
C-Programming  
Semester -I: End Examination

Time: 3 Hours

Max. Marks: 60

**PART A**

**I. Answer All Multiple Choice Questions:**

**15X1=15M**

1. C programming language was developed by  
a) Dennis Ritchie      b) Ken Thompson      c) Bill Gates      d) Peter Norton
2. C is a \_\_\_ language  
a) High Level      b) Low Level      c) Middle Level      d) Machine Level
3. Which of the following symbol is used to denote a pre-processor statement?  
a) !      b) #      c) ~      d) ;
4. Which of the following is a Scalar Data Type?  
a) Float      b) Union      c) Array      d) Pointer
5. What is the valid range of numbers for int type of data?  
a) 0 to 256      b) -32768 to +32767      c) -65536 to +65536      d) No specific range
6. Which escape character can be used to begin a new line in C?  
a) \a      b) \b      c) \m      d) \n
7. String constants should be enclosed between \_\_\_\_  
a) Single quotes      b) Double quotes      c) Both a and b      d) None of these
8. The maximum length of a variable in C is \_\_\_\_  
a) 8      b) 16      c) 32      d) 64
9. Pointers are of  
a) Integer data type      b) character data type      c) unsigned integer data types      d) None of these
10. Which is the correct way to declare a pointer?  
a) int\_ptr;      b) int \*ptr;      c) \*int ptr;      d) None of these.
11. Which of the following is an example of compounded assignment statement?  
a) a=5      b) a+=5      c) a=b=c      d) a=b
12. The process of translating a source program into machine language is a function of:  
a) Compiler      b) Translator      c) Assembler      d) None of these.
13. Operators have hierarchy. It is useful to know which operator  
a) is most important      b) is used first      c) is faster      d) operates on large numbers
14. C supports the \_\_\_\_\_ statement to branch unconditionally from one point to another in the program.  
a) Continue      b) goto      c) break      d) for
15. The \_\_\_\_\_ is used to break out of the case statements.  
a) Continue      b) break      c) default      d) case

PART B

5x9=45 M

II. Answer any one question out of two questions from each unit

16. a. Explain various types of data types that are available in C Language.  
Or

b. What is a variable? What are the rules that are to be followed during variable declaration?

17 a. What is Control Statements with examples?

Or

b. What is case control statement & jump statements with examples?

18. a. What is meant by an array? Explain uses and types of an array?  
Or

b. Explain Character and String functions.

19. a . What is meant by the scope of variables? Explain storage classes in 'C'?  
Or

b. Explain (i) Pointer and Arrays (ii) Passing arrays to functions (iii) Difference between Array Name and Pointer.

20. a . What is a structure and explain the rules for declaration, initialization and accessing structure?  
Or

b. What is a file? Explain.

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**(PMAT 103)**  
**M.Sc (Mathematics) P.G Examinations-July 2022**  
**SEMESTER-I**  
**C-PROGRAMMING**

TIME: 3Hrs

MaxMarks:60

**SECTION - A**

**Answer any FIVE Questions out of the 10 Short Answer Questions 5 x 4 = 20 M**

1. Brief the history of C Programming Language
2. Discuss various Data types available in C
3. Differentiate 'break' and 'continue' statements with an example
4. Write the syntax of 'switch' statement
5. Define Array. Explain how to initialize array with example
6. Write a C Program to read and print a two dimensional Array
7. Write a short note on null pointer and generic pointer
8. Define Function. Explain various uses of functions
9. Explain the concept of nested structures with an example
10. Explain various types of file modes

**SECTION - B**

Answer the following Questions

**5 x 8 = 40 M**

11a) Explain the structure of a C program with an example  
(OR)

b) Explain various types of operators available in C

12a) Explain various looping control statements with examples  
(OR)

b) Explain various decision control statements with syntax and examples

13a) Write a C Program to accept array of numbers and find out largest and smallest among them

(OR)

b) Discuss various String handling functions with examples

14 a) Explain differences between call by value and call by reference  
(OR)

b) Explain different types of functions with examples

15 a) Differentiate between Structures and Unions  
(OR)

b) Discuss about various file operations

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Part - A

I. Answer all multiple choice questions  $15 \times 1 = 15M$

① If  $G$  is a Group, then for all  $a, b \in G$

(a)  $(ab)^{-1} = a^{-1}b^{-1}$  (b)  $(ab)^{-1} = b^{-1}a^{-1}$  (c)  $(ab)^{-1} = ab$

(d)  $(ab)^{-1} = ba$

② If  $G$  is a group of order  $n$  then, order of identity element is

(a)  $n$  (b) Greater than one (c) one (d) None

③ If the orders of elements  $a, a^{-1} \in G$  are  $m$  and  $n$  respectively then

(a)  $m = n$  (b)  $m \neq n$  (c)  $m = n = 0$  (d) None

④ If given permutations are  $A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$

$B = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & 5 & 2 \end{pmatrix}$  then  $BA$  is

(a)  $\begin{pmatrix} 2 & 1 & 5 & 3 & 5 \\ 1 & 6 & 4 & 2 & 1 \end{pmatrix}$  (b)  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & 4 & 1 \end{pmatrix}$

(c)  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}$  (d)  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 5 & 4 & 3 \end{pmatrix}$

⑤ The Identity permutation is —

(a) odd permutation (b) Even permutation

(c) Neither even nor odd (d) None of these.

⑥ The permutation  $\begin{pmatrix} 1 & 2 & 5 & 3 & 4 \\ 3 & 4 & 1 & 5 & 2 \end{pmatrix}$  is equal to

(a) (1) (2) (3)

(b) (1 3 5) (5 6)

(c) (1 3) (1 5) (2 4)

(d) (1 4 2) (5 3)

⑦ If  $p$  is a prime number and  $p \mid o(G)$ , then  $a \in G$  such that

(a)  $a^p \notin G$  (b)  $a^p \in G$  (c)  $a^p \in G$  (d)  $G \subset a^p$ .

⑧ If order of a group  $G$  is  $p^2$ , where  $p$  is prime then

(a)  $G$  is a ring (b)  $G$  is abelian (c)  $G$  is not abelian (d) None of these

⑨ If  $a, b \in G$ , a group then  $b$  is conjugate to  $a$  if there exist  $c \in G$  such that

(a)  $a = cb$  (b)  $b = a\bar{c}$  (c)  $b = \bar{c}ac$

(d)  $b = c\bar{c}a$ .

⑩  $E$  is the set of even integers under ordinary addition and multiplication, then  $E$  is a ring

$E$  is also a

(a) Field (b) Integral domain (c) Commutative Ring

(d) None of these.

11. The non zero elements  $a, b$  of a ring  $(R, +, \cdot)$  are called zero divisors if

- (a)  $a \cdot b = 1$  (b)  $a \cdot b \neq 0$  (c)  $a \cdot b = 0$  (d)  $a \cdot b \neq 1$

12. The following statement is false

- (a) Every Field is an integral domain  
(b) Every integral domain is a field  
(c) Every field is a Ring  
(d) Every ring is a group.

13. A polynomial  $f(x)$  and  $g(x)$  are primitive polynomials then

- (a)  $f(x) + g(x)$  is a primitive polynomial  
(b)  $f(x) - g(x)$  is a primitive polynomial  
(c)  $f(x) \cdot g(x)$  is a primitive polynomial  
(d) None of these

14. which of the following statement is false

- (a)  $F[x]$  is an integral domain  
(b)  $F[x]$  is Euclidean ring  
(c)  $F[x]$  is principal ideal ring  
(d)  $F[x]$  is not a group.

15. If  $f(x)$  and  $g(x)$  are two non zero polynomials of  $F[x]$  then

- (a)  $\deg(f(x) \cdot g(x)) = \deg f(x) + \deg g(x)$   
(b)  $\deg(f(x) \cdot g(x)) = \deg f(x) \cdot \deg g(x)$

(c)  $\deg (f(x) \cdot g(x)) = \deg f(x) + \deg g(x)$

(d) None of these.

Part - B

II Answer the following questions

3x9 = 45M

16) a) State and prove Lagrange's theorem. Is the converse true. Justify your answer.

(OR)

b) If  $G$  is a group, then prove that  $A(G)$ , the set of automorphism of  $G$  is also a group.

17) a) prove that the set  $A_n$  of all even permutations on  $n$  symbols is a normal subgroup of permutation group  $S_n$  of  $n$  symbols and

$|A_n| = \frac{1}{2} (n!)$  (OR)

b) state and prove Cayley's theorem

18) a) state and prove Sylow's first theorem (OR)

b) prove that if  $p$  is a prime number such that  $p$  divides  $|G|$ , then  $G$  has an element of order  $p$ .

19) a) prove that  $\mathbb{Z}$  is a principal ideal Ring (OR)

b) prove that every integral domain can be embedded in a field.

a) state and prove Eisenstein criterion for irreducibility.

(OR)

b) If  $R$  is an Integral domain, then prove that  $R[x]$  is an Integral domain.

————— x x x x —————

**(PMAT 104)**  
**M.Sc (Mathematics) P.G Examinations-July 2022**  
**SEMESTER-I**  
**ALGEBRA**

TIME: 3Hrs

MaxMarks: 60

SECTION-A

Answer any **FIVE** of the following. Each question carries 4 Marks. **5 x 4 = 20 M**

1. If  $a^2 = e \forall a \in G$ , then prove that  $G$  is abelian.
2. Define kernel of  $\phi$ . Let  $\phi$  be a group homomorphism of  $G$  into  $G^1$ .  
Then prove that kernel of  $\phi$  is a normal subgroup of  $G$ .
3. Prove that every group of order  $p^2$  is abelian.
4. Define Normalizer of an element and prove that  $N(a)$  is a subgroup of  $G$ .
5. Show that the group of order 56 is not simple.
6. Define external direct product of groups.
7. Prove that a field has no proper ideals.
8. Let  $R$  be a commutative ring with unit element whose only ideals are  $\{0\}$  and  $R$  itself. Then prove that  $R$  is a field.
9. Define ring of polynomials, Euclidean Domain, Principle Ideal Domain, Unique Factorization Domain.
10. State and prove Fermat's theorem.

## SECTION-B

Answer ALL questions. Each question carries 8 Marks.

5 x 8 = 40 M

(1) a) State and prove the Lagrange's theorem for finite groups. Is the converse true? Justify.

(OR)

b) Define automorphism of a group. Prove that if  $G$  is a group, then  $A(G)$ , the set of automorphisms of a group  $G$  is also a group.

(2) a) Let  $G$  be a group. Then show that the following are true :

(i) The set of conjugate classes of  $G$  is a partition of  $G$

(ii)  $|C(a)| = [G:N(a)]$

(iii) If  $G$  is finite,  $|G| = \sum [G:N(a)]$ , a running over exactly one element from each conjugate class.

(OR)

b) State and prove Cayley's theorem.

(3) a) State and prove Sylow's 1<sup>st</sup> and 2<sup>nd</sup> theorems.

(OR)

b) State and prove fundamental theorem of finitely generated abelian groups.

(4) a) If  $R$  is a ring with unity, then prove that each maximal ideal is prime.

But the converse need not be true.

(OR)

b) Define embedded of ring. Prove that every integral can be embedded into a field.

(5) a) Prove that every principle ideal domain is a Unique Factorization Domain.

(OR)

b) State and prove Gauss Lemma.

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